New conditional symmetries and exact solutions of nonlinear reaction-diffusion-convection equations. III

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A complete description of Q-conditional symmetries of reaction-diffusion-convection equation with arbitrary power nonlinearities is finished. It is shown that the results obtained in the first and second parts of this work (see arXiv: math-ph/0612078 and arXiv: math-ph/0706.0814) cannot be extended on new power nonlinearities arising in the diffusion and convection coefficients.

1. Introduction.

In the paper [1] (see [2]-[3] for details) the complete description of Q-conditional symmetries of reaction-diffusion-convection equations

$$U_t = [U^m U_x]_x + \lambda U^m U_x + C(U), \tag{1}$$

$$U_t = [U^m U_x]_x + \lambda U^{m+1} U_x + C(U), \tag{2}$$

where λ and m are arbitrary constants and $C(U)$ is an arbitrary smooth function, has been done. The symmetries obtained for constructing exact solutions of the relevant equations have been successfully applied. In the particular case, new exact solutions of nonlinear reaction-diffusion-convection (RDC) equations arising in applications have been found.

The most general RDC equation with power functions arising in the diffusion and convection coefficients reads as

$$U_t = [U^m U_x]_x + \lambda U^n U_x + C(U). \tag{3}$$

In the cases $n = m$ and $n = m + 1$, this equation coincides with (1) and (2), respectively. Here we report the main result concerning the structure of Q-conditional symmetries of equation (3). Note that equation (3) with $n = 0$ is reducing by local substitution $y = x + \lambda t$ to the equation

$$U_t = (U^m U_y)_y + C(U).$$

Q-conditional symmetries of this reaction-diffusion equation were investigated in [4], so that we assume $n \neq 0$.

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2. Main Result.

This is well-known that a (1+1)-dimensional evolution equation may admit Q-conditional symmetries of two different forms

$$Q = \partial_t + \xi(t,x,U)\partial_x + \eta(t,x,U)\partial_U, \tag{4}$$

and

$$Q = \partial_x + \eta(t,x,U)\partial_U, \tag{5}$$

where ξ and η are unknown smooth functions, which should be found.

We have proved that equation (3) with $\lambda n \neq 0,\ n \neq m, m+1$ cannot admit any new Q-conditional operators of the form (4). In other words, a RDC equation with power coefficients of diffusion and convection admits Q-conditional symmetries of the form (4) only in the cases presented in [1].

Theorem 1 *The RDC equation (3) with $\lambda n \neq 0,\ n \neq m, m+1$ is invariant only with respect to operators of the form (4), which are equivalent to a linear combination of the Lie symmetry operators listed in table 1 of [5].*

It should be stressed that we didn't consider the problem of constructing Q-conditional symmetries of the form (5) because one is equivalent (up to the known non-local transformation) to solving the given equation (3) [6]. Obviously, the nonlinear RDC equation (3) is not integrable therefore a complete description of Q-conditional symmetries of this form cannot be derived. On the other hand, one can try to find *particular* solutions of the relevant determining equation for the function η, which was derived in [7], and to construct some operators of the form (5).

To prove theorem 1 we use the substitution [2]

$$V = \begin{cases} U^{m+1},\ m \neq -1, \\ \ln U,\ m = -1. \end{cases} \tag{6}$$

In the case $m \neq -1$ substitution (6) reduces equation (3) to the form

$$V_{xx} = V^p V_t - \lambda V^k V_x + F(V),\ \lambda \neq 0;\ p \neq -1;\ k \neq 0, p, p+1, \tag{7}$$

where $p = -\frac{m}{m+1},\ k = \frac{n-m}{m+1},\ F(V) = -(m+1)C\left(V^{\frac{1}{m+1}}\right),$ and in the case $m = -1$ to the form

$$V_{xx} = e^V V_t - \lambda e^{(n+1)V} V_x + F(V),\ \lambda \neq 0,\ n \neq 0, -1 \tag{8}$$

where $F(V) = -C(e^V)$. We use the work [7] to obtain the system of determining equations for finding the coefficients of the operator

$$Q = \ \partial_t + \xi(t,x,V)\partial_x + \eta(t,x,V)\partial_V, \tag{9}$$

which is locally equivalent to the operator (4) (up to notations).

In the case $F_0(V)=V^p,\ F_1(V)=-\lambda V^k,\ F_2(V)=F(V)$, system (2.38) [7] takes the form

$$\begin{aligned}
&\xi_{VV}=0,\\
&\eta_{VV}=2\xi_V(-\lambda V^k-\xi V^p)+2\xi_{xV},\\
&(2\xi_V\eta-2\xi\xi_x-\xi_t)V^p-\xi\eta pV^{p-1}-\lambda\xi_xV^k-\lambda k\eta V^{k-1}+\\
&+3\xi_VF-2\eta_{xV}+\xi_{xx}=0,\\
&\eta F_V+(2\xi_x-\eta_V)F+(2\xi_x\eta+\eta_t)V^p+p\eta^2V^{p-1}-\\
&-\lambda\eta_xV^k-\eta_{xx}=0,
\end{aligned}\tag{10}$$

and in the case $F_0(V)=e^V,\ F_1(V)=-\lambda e^{(n+1)V},\ F_2(V)=F(V)$, one takes the form

$$\begin{aligned}
&\xi_{VV}=0,\\
&\eta_{VV}=2\xi_V(-\lambda e^{(n+1)V}-\xi e^V)+2\xi_{xV},\\
&(\xi_t+2\xi\xi_x+(\xi+\lambda(n+1)-2\xi_V)\eta)e^V+\\
&+\lambda\xi_xe^{(n+1)V}-3\xi_VF+2\eta_{xV}-\xi_{xx}=0,\\
&\eta F_V+(2\xi_x-\eta_V)F+\eta^2e^V+2\xi_x\eta e^V+\eta_te^V-\\
&-\lambda\eta_xe^{(n+1)V}-\eta_{xx}=0.
\end{aligned}\tag{11}$$

We divide the solving systems (10) and (11) on three cases:

$(a)\ \xi=aV+b,\ \eta=\eta(V),\ a=const, b=const$

$(b)\ \xi=a(t,x)V+f(t,x),\ a(t,x)\neq 0,$

$(c)\ \xi=f(t,x),\ \eta=g(t,x)V+h(t,x).$

One can easily check that these cases take into account all possible solutions of the systems (10) and (11).

Theorem 2 *In the cases* (*a*) *and* (*b*) *equations (7) and (8) can be invariant only with respect to the Lie symmetry operator of the form*

$$Q=\partial_t+\xi\partial_x,\ \xi=const.\tag{12}$$

Theorem 3 *In the case* (*c*) *equations (7) and (8) are invariant only with respect to operators of the form (9), which are equivalent to the Lie symmetry operators listed in [7] and [5].*

Theorem 1 immediatelly follows from theorems 2 and 3 if one takes into account that the RDC equation (3) is locally equivalent to the equations (7) (if $m\neq-1$) and (8) (if $m=-1$).

3. Proof of Theorem 2

Firstly, let us consider the case (a). In the quite similar way as it was done in [2] (see pp. 11–14) one proves that operator (4) may take only form (12), which is the Lie symmetry operator of the equations (7) and (8).

Case (b). Let us consider the system (10) (the consideration of the system (11) is quite similar). The general solution of the first equation arising in (10) is

$$\xi = a(t,x)V + f(t,x), \tag{13}$$

the solution of the second equation of (10) is

$$\begin{aligned} \eta = &-\tfrac{2a^2}{(p+2)(p+3)}V^{p+3} - \tfrac{2af}{(p+1)(p+2)}V^{p+2} - \tfrac{2\lambda a}{(k+1)(k+2)}V^{k+2} + \\ &+a_x V^2 + g(t,x)V + h(t,x), \end{aligned} \tag{14}$$

if $p \neq -2,-3,\ k \neq -1,-2$ (the consideration of the cases $p=-2,-3,\ k=-1,-2$ is much more simpler). Substituting (14) into the third equation of system (10) we obtain

$$\begin{aligned} F(V) = \quad & \frac{1}{3}\Bigg(-\frac{2a^2(p-2)}{(p+2)(p+3)}V^{2p+3} - \frac{2pf^2}{(p+1)(p+2)}V^{2p+1} - \\ & -2\lambda a\left(\frac{k}{(p+2)(p+3)} + \frac{p-2}{(k+1)(k+2)}\right)V^{p+k+2} - \\ & -2\lambda f\left(\frac{k}{(p+1)(p+2)} + \frac{p}{(k+1)(k+2)}\right)V^{k+p+1} - \\ & -4af\frac{p^2+p-3}{(p+1)(p+2)(p+3)}V^{2p+2} + a_x\frac{(p-2)(p+4)}{p+2}V^{p+2} + \\ & +\frac{1}{a}\bigg(\Big(2(af_x + a_x f)\frac{p-1}{p+1} + a_t + pfa_x + ag(p-2)\Big)V^{p+1} + \\ & +(f_t + 2ff_x + pfg + ah(p-2))V^p + pfhV^{p-1} + \\ & +\lambda a_x\frac{(k-1)(k+3)}{k+1}V^{k+1} + \lambda(f_x + kg)V^k + \lambda khV^{k-1} + \\ & +3a_{xx}V + 2g_x - f_{xx}\bigg) - \frac{2\lambda^2 k}{(k+1)(k+2)}V^{2k+1}\Bigg). \end{aligned} \tag{15}$$

Since the function F depends only on the variable V, all coefficients by different powers of this variable must be constants. In the general case, we obtain fifteen equations for determining the functions $a,\ f,\ g,\ h$. The number of equations may be shortened for certain values of k and p .

It turns out that the functions g and h must be constants, if $a(t,x) = const,\ f(t,x) = const$ therefore we arrive at the case (a), which lead only to the Lie operator (12). Let us prove this statement.

Expression (15) with $a = const,\ f = const$, takes the form

$$\begin{aligned} F = &\frac{1}{3}\Big(M(V) + \frac{1}{a}\Big(ag(p-2)V^{p+1} + (pfg + ah(p-2))V^p + \\ &+pfhV^{p-1} + \lambda kgV^k + \lambda khV^{k-1} + 2g_x\Big)\Big), \end{aligned} \tag{16}$$

where

$$M(V) = -\frac{2a^2(p-2)}{(p+2)(p+3)}V^{2p+3} - \frac{2pf^2}{(p+1)(p+2)}V^{2p+1} -$$
$$-2\lambda a\left(\frac{k}{(p+2)(p+3)} + \frac{p-2}{(k+1)(k+2)}\right)V^{p+k+2} -$$
$$-2\lambda f\left(\frac{k}{(p+1)(p+2)} + \frac{p}{(k+1)(k+2)}\right)V^{k+p+1} -$$
$$-4af\frac{p^2+p-3}{(p+1)(p+2)(p+3)}V^{2p+2} - \frac{2\lambda^2 k}{(k+1)(k+2)}V^{2k+1}$$

is the polynomial, which depends only on V. Analyzing powers $p+1,\ p,\ p-1,\ k,\ k-1,\ 0$ in (16), we receive the conclusion that there are only five cases, when two and more among them are equal, namely:

$$k = p-1,\ k = p+2,\ p = 0,\ p = 1,\ k = 1. \tag{17}$$

We have to investigate also the general case, when conditions (17) don't take place. In the general case, all coefficients by the powers of V must be constant. Let us consider the coefficients by the terms V^{p+1} : $\dfrac{1}{3}g(p-2) = const$ and V^p : $\dfrac{1}{3a}(pfg + ah(p-2)) = const$. Considering separately two subcases, $p \neq 2$ and $p = 2$ one easily obtains that $g(t,x) = const,\ h(t,x) = const$, provided $a(t,x) = const,\ f(t,x) = const$.

Let us consider the case $k = p-1$ from (17) leading to the powers $p+1,\ p,\ p-1,\ p-1,\ p-2,\ 0$ of the variable V in (16). We must separately consider the subcases $p = 2, p = -1, p = 0, p = 1$ and $p \neq 2, 1, 0, -1$.

The expression (16) with $p = 2,\ k = p-1 = 1$ gets the form

$$F = \frac{1}{3}\Big(M(V) + \frac{1}{a}\Big(2fgV^2 + (2fh + \lambda g)V + h + 2g_x\Big)\Big). \tag{18}$$

Since the right-hand-sight of (18) cannot depend on $t,\ x$, we immediately obtain $g = const$ and $h = const$.

In the case $p \neq 2, 1, 0, -1,\ k = p-1$ expression (16) takes the form

$$F = \frac{1}{3}\Big(M(V) + \frac{1}{a}\Big(ag(p-2)V^{p+1} + (pfg + ah(p-2))V^p +$$
$$+ pfhV^{p-1} + \lambda(p-1)gV^{p-1} + \lambda(p-1)hV^{p-2} + 2g_x\Big)\Big).$$

Since coefficients by V^{p+1} and V^p must be constant we again obtain $g = const$ and $h = const$. Subcase $p = -1$ contradicts to the condition presented above in (7), while the subcases $p = 0$ and $p = 1$ should be considered for arbitrary k (see (17)).

We have checked other cases from (17) and obtained the same result. Thus, the functions g and h are some constants in the expression (15) provided $a = const,\ f = const$.

On the other hand, the functions a and f are constants if the powers $2p+3$ and $2p+1$ are not equal to any other power in (15).

Thus, to prove theorem 2 we must consider only such cases, when the powers $2p+3$ and $2p+1$ are equal to other power(s) in (15). One can easily check that this happens only in the following cases:

$$\begin{aligned}&1)\ p = -4,\ 2)\ p = -\tfrac{3}{2},\ 3)\ p = -\tfrac{1}{2},\ 4)\ p = 0,\ 5)\ p = 1,\ 6)\ p = 2,\\ &7)\ k = p-1,\ 8)\ k = p+2,\ 9)\ k = 2p,\ 10)\ k = 2p+1,\ 11)\ k = 2p+2,\\ &12)\ k = 2p+3,\ 13)\ k = 2p+4.\end{aligned} \tag{19}$$

Let us consider the case 1) from (19) in details (all other cases can be investigated in a quite similar way). Consider fifteen powers of the variable V, arising in (15) with $k = p - 1$: $2p + 3, 2p + 1, 2p + 1, 2p, 2p + 2, p + 2, p + 1, p, p - 1, p, p - 1, p - 2, 1, 0, 2p - 1$. Let us form the table 1 with those values of p listed in the second row, when at least one of the powers listed in the first row is equal to $2p+3$ (we don't write down the power $2p-1$ because the corresponding coefficient is already constant).

Table 1.

	$2p+1$	$2p$	$2p+2$	$p+2$	$p+1$	p	$p-1$	$p-2$	1	0
$2p+3$	-	-	-	-1	-2	-3	-4	-5	-1	$-\frac{3}{2}$

The values $p = -1, -2, -3$ contradict to the conditions presented above (see (7), (14)). Other values p listed in the second row of table 1 are only subcases of the corresponding cases from (19), namely: $p = -\frac{3}{2}$ and $p = -4$ are subcases of cases 7) and 11), respectively, $p = -5$ lead to $k = -6$, so that this is a subcase of 8).

Thus, the power $2p + 3$ doesn't coincide with any other, so that, $a = const$ (see the first term in right-hand-side of (15)). Otherwise we should put $p = 2$ but this is another case from (19).

To establish that $f = const$ we analyze the power $2p + 1$. Let us form the table 2 in the same way as we built table 1 and taking into account that $a = const$. We obtain five special values of parameter p.

Table 2.

	$2p$	$2p+2$	$p+1$	p	$p-1$	$p-2$	0
$2p+1$	-	-	0	-1	-2	-3	$-\frac{1}{2}$

Again the values $p = 0, -1, -2, -3$ contradict to conditions presented above (see (7), (14)). The last value $p = -\frac{1}{2}$ listed in the second row of table 2 is only a subcases of case 3) from (19). This means that the power $2p+1$ doesn't coincide with any other power, hence, $f = const$ (see the second term in right-hand-side of (15)).

Thus, we obtain that $a = const$ and $f = const$ in the case 1) from (19), so that, taking into account the statement proved above, we arrive at the case (a).

All other cases from (19) have been analyzed a similar way and the same result established, i.e. $a = const$ and $f = const$.

The proof is now completed.

4. Sketch of the Proof of Theorem 3

Substituting ξ and η from the case (c) into the third equation of (10) (the investigation of system (11) is analogous), we obtain

$$(2ff_x + f_t + pfg)V^p + pfhV^{p-1} + \lambda(f_x + kg)V^k + \lambda khV^{k-1} + 2g_x - f_{xx} = 0. \tag{20}$$

To analyze (20) one needs to consider only the following special cases: $p = 0,\ p = 1,\ k = p - 1,\ k = 1$ and the general case, when p and k don't satisfy these restrictions.

Consider the case $p=0$ in detail (the next two cases and the general case are investigated in the same way). Equation (20) with $p=0$ takes the form

$$\lambda(f_x+kg)V^k+\lambda khV^{k-1}+f_t+2ff_x-f_{xx}+2g_x=0. \tag{21}$$

Since $k\neq p=0$ and $k\neq p+1=1$, one can split (21) by different powers of V and obtain the system

$$\begin{aligned}&\lambda kh=0,\\&\lambda(f_x+kg)=0,\\&f_t+2ff_x-f_{xx}+2g_x=0.\end{aligned} \tag{22}$$

Taking into account the restrictions presented above, we obtain

$$h=0, f_x=-kg. \tag{23}$$

Substituting expressions (23) into the fourth equation of (10), we arrive at the linear ODE

$$gVF_V-(2k+1)gF=\lambda g_xV^{k+1}+(g_{xx}+2kg^2-g_t)V \tag{24}$$

with the general solution

$$F=\lambda_1V^{2k+1}-\frac{\lambda g_x}{kg}V^{k+1}+\Big(\frac{g_t-g_{xx}}{2kg}-g\Big)V. \tag{25}$$

Note the special subcase $g=0$ immediately leads to $h=0$ and $f=const$, therefore the Lie symmetry (12) is obtained.

Since right-hand-side of (25) cannot depend on the independent variables, we obtain

$$\begin{aligned}&\tfrac{\lambda g_x}{kg}=\lambda_2,\\&\tfrac{g_t-g_{xx}}{2kg}-g=\lambda_3,\end{aligned} \tag{26}$$

where lambda-s are some constants. The general solution of (26) is $g=\alpha(t)$ and then, using (23), we have

$$f=-k\alpha(t)x+\beta(t), \tag{27}$$

where $\alpha(t)$, $\beta(t)$ are to-be-determined functions. Substituting (27) into the third equation of (22), we obtain

$$(-k\alpha_t+2k^2\alpha^2)x+\beta_t-2k\alpha\beta=0. \tag{28}$$

Obviously, equation (28) is equivalent to two ODE equations with the general solution

$$\alpha=-\frac{1}{2kt+A_1},\ \beta=\frac{A_2}{2kt+A_1},\ A_i=const,\ i=1,2. \tag{29}$$

Finally, taking into account (23), (25), (27) and (29), we have found that equation

$$V_{xx}=V_t-\lambda V^kV_x+\lambda_1V^{2k+1},$$

is invariant under the operator

$$Q = \partial_t + \frac{1}{2kt + A_1}\Big((kx + A_2)\partial_x - V\partial_V\Big).$$

Multiplying this operator by $2kt + A_1$ one obtains the operator

$$X = (2kt + A_1)\partial_t + (kx + A_2)\partial_x - V\partial_V, \tag{30}$$

which is nothing else but a linear combination of Lie symmetry operators of this equation (see the case 7 of table 1 [5]).

Let us consider the case $k = 1$, which is special. Equation (20) with $k = 1$ takes the form

$$(2ff_x + f_t + pfg)V^p + pfhV^{p-1} + \lambda(f_x + g)V + \lambda h + 2g_x - f_{xx} = 0. \tag{31}$$

We must consider only two subcases $p \neq 2$ and $p = 2$ (we remind the reader that the subcases $p = 0$ and $p = 1$ were considered above). It is easily shown that the first of them leads only to Lie symmetries, since we can split the equation (31) with respect to the four different powers of V, i.e., four equations are obtained. In the second subcase (31) takes the form

$$(2ff_x + f_t + 2fg)V^2 + (2fh + \lambda(f_x + g))V + \lambda h + 2g_x - f_{xx} = 0 \tag{32}$$

and we obtain only three equations

$$\begin{aligned} &2ff_x + f_t + 2fg = 0,\\ &2fh + \lambda(f_x + g) = 0,\\ &\lambda h + 2g_x - f_{xx} = 0. \end{aligned} \tag{33}$$

So, (32) is the nonlinear system of three PDEs for three unknown functions.

We must also take into account the fourth equation of (10) with $p = 2$, namely:

$$\begin{aligned} &(gV + h)F_V + (2f_x - g)F = -(gV + h)(3gV^2 + 2hV) -\\ &-(2f_x - g)(gV + h)V^2 - (g_tV + h_t)V^2 + \lambda(g_xV + h_x)V + g_{xx}V + h_{xx}. \end{aligned} \tag{34}$$

Analyzing the first term of (34) we must consider three subcases, 1) $g = h = 0$, 2) $g = 0,\ h \neq 0$, 3) $g \neq 0$, for solving this equation. It turns out, subcases 1) and 2) lead the Lie symmetry (12) (we omit the relevant calculations because they are rather simple).

Let us consider the most complicated case 3) $g \neq 0$. The general solution of the equation (34) is

$$F = \lambda_3V^3 + \lambda_2V^2 + \lambda_1V + \lambda_0 + A\Big(V + \frac{h}{g}\Big)^{1-\frac{2f_x}{g}},\ A = const. \tag{35}$$

Here $\lambda_i,\ i = 0, 1, 2, 3$ are some functions on $f,\ g,\ h$ and their derivatives but we omit those expressions for lambda-s because they are rather cumbersome. Since the function F must depend only on V, then we obtain

$$1 - \frac{2f_x}{g} = A_1^*, \quad A_1^* = const \tag{36}$$

if $A \neq 0$. Formula (36) may be written in the form

$$g = A_1 f_x, \quad A_1 = \frac{2}{1 - A_1^*} \tag{37}$$

(the case $A_1^* = 1$ leads to the contradiction $g = 0$). Substituting (37) into the third equation of (33) we obtain

$$h = \frac{1 - 2A_1}{\lambda} f_{xx}. \tag{38}$$

Substituting (37) and (38) into the second equation of (33), we arrive at

$$2(2A_1 - 1) f f_{xx} = \lambda^2 (A_1 + 1) f_x. \tag{39}$$

Equation (39) is reduced by the substitution $y(f) = f_x$ to the form

$$f_x = \frac{\lambda^2 (A_1 + 1)}{2(2A_1 - 1)} \ln(\gamma(t) f), \tag{40}$$

where $\gamma(t)$ is an arbitrary smooth function (the case $A_1 = \frac{1}{2}$ leads again to the contradiction $g = 0$). Substituting (40) into the first equation of (33) and taking into account (37), we obtain

$$f_t = -\frac{\lambda^2 (A_1 + 1)^2}{2A_1 - 1} f \ln(\gamma f). \tag{41}$$

Differentiating (40) by t and (41) by x and equaling the expressions obtained, we have

$$\frac{\lambda^2 (A_1 + 1)}{2A_1 - 1} \left(\frac{\gamma_t}{2\gamma} + \frac{\lambda^2 (A_1 + 1)^2}{2(2A_1 - 1)} (\ln(\gamma f))^2 \right) = 0. \tag{42}$$

The expression (42) is satisfying only with $A_1 = -1$, but this leads to $f_x = 0$ (see (40)) and therefore $g = 0$ (see (37)). Thus, the contradiction is again obtained.

Let us consider (35) with $A = 0$. Substituting (35) with $A = 0$, into equation (34) and splitting expression obtained by the different powers of V, we obtain the system

$$\begin{aligned}
&g_t + 2(g + \lambda_3)(g + f_x) = 0, \\
&2f_x(h + \lambda_2) + g(4h + \lambda_2) + 3\lambda_3 h + h_t - \lambda g_x = 0, \\
&2h(h + \lambda_2) + 2\lambda_1 f_x - \lambda h_x - g_{xx} = 0, \\
&\lambda_1 h + 2\lambda_0 f_x - \lambda_0 g - h_{xx} = 0.
\end{aligned} \tag{43}$$

Finally, any solution of the nonlinear system (33) and (43), which consist of seven equations on three functions $f,\ g$ and h generate the operator of Q-conditional symmetry

$$Q = \partial_t + f\partial_x + (gV + h)\partial_V \tag{44}$$

for the equation

$$V_{xx} = V^2 V_t - \lambda V V_x + \lambda_3 V^3 + \lambda_2 V^2 + \lambda_1 V + \lambda_0. \tag{45}$$

It turns out, that the overdetermined system of PDEs (33) and (43) is compactable. However, all its solutions produce the operators of the form (44), which are nothing else but Lie symmetry operators of (45) obtained in [7]. We have established this using computer algebra package Mathematica 5.0. The relevant calculations are omitted because their awkwardness.

The sketch of the proof of theorem 2 is now completed.

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